

Kurdistan Regional government Kurdistan Engineering Union

Design of Steel Structure Research on Steel and Composite Bridge Design Basic principles

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INTRODUCTION TO MODERN COMPOSITE BRIDGES

Composite bridges use the compressive resistance of concrete and the tensile resistance of steel to greatly increase the bending resistance and stiffness of the steel beams, which improves its span capabilities. Steel is attractive for use in composite bridges because:

- A wide range of rolled or fabricated sections can be used
- Speed of installation means that road closures and disruption are minimised
- Steel bridges are lightweight (less than half of the equivalent concrete bridges)
- Composite bridges are very efficient structurally
- Steel girders can be fabricated as asymmetric sections for improved performance as composite sections
- Continuous bridges can be designed by continuity of the reinforcement in the slab
- "Box" sections can be designed compositely and are good in resisting torsion for use in curved bridges.
- Steel can be easily repaired, maintained and re-used or re-cycled
- The proven life span of steel bridges is over 1++ years.

For motorway bridges, multi beam bridges are often used for spans of $7 \cdot m$ to $2 \cdot m$. The beams may be parallel, or skewed or curved in-plane. For spans of $7 \cdot$ to $7 \circ m$, rolled Universal Beams up to $1 \cdot 17 \times 7 \cdot 0 \times 2 \wedge V$ kg/m may be used, but fabricated steel beams are generally used for long spans.

Fabricated beams

Fabricated beams can be made asymmetric by use of flanges of different sizes in order to optimise their bending properties (the bottom flange is usually larger than the top flange). Webs can be slender and stiffened, but it is usually more efficient to use thicker plates to avoid stiffening (except locally at bearings). For reasons of transport, the maximum length of a beam or component is 70 to $7 \cdot m$, and so it is often required to splice a beam at the points of low moment (such as the quarter span positions).

The size of the plates of fabricated beams is usually based on the following approximate limits:

Depth of web of beam =	Span of beam /۲o to /۲۰
Width of flange =	•, \tilde{r} to •, ε x Depth of beam
Thickness of web =	Depth of web/٦٠ to ∧٠
Thickness of flange =	Width of flange/۲۰ to ۳۰
Asymmetry in flange areas =	1,0 to 7,0 typically

The web of fabricated beams can be cut to create a pre-camber in the beam to off-set deflections.

Box girders

Box girders are formed in "box" shape from plate and may be "open-topped" or "closed" in cross-section. They may be simple beams or in pairs (i.e. below carriageways) and may be of variable cross-sections and curved in-plain. Usually the plates are thin and lightweight and are therefore highly stiffened to create an orthotropic plate (particularly for the top plate). Box girders are widely used in suspension bridges.

Continuous bridges

Continuity can be achieved through the steel section and reinforcement in the slab, so that negative (hogging) moments can be resisted. In this way, the beams can be lighter and more slender, and movement joints in the bridge wearing surface are avoided. However, the reinforcement is concentrated in the slab and some congestion of the bars may result.

Pre-cast bridge decks

Pre-fabricated concrete slabs may be used to replace in-situ concrete slabs to speed up the construction. "Pockets" are left in the pre-cast slab for the groups of shear connectors that are pre-welded to the beam. Transverse reinforcement is also concentrated in this zone.

Shear connectors

Shear connectors are usually welded to the bridge beam before delivering to site. They are generally in the form of welded headed "studs" of 19 to 72mm diameter and 1++ to 7++mm height. In some cases, channel sections can be welded to the beam. Shear connectors transfer the longitudinal forces between the steel and concrete. They can be spaced to suit the shear flow, and can be "grouped" together. Transverse reinforcement is required to transfer the high local forces into the slab.

Innovations

A variety of innovations may be possible within the design concept for composite bridges such as use of:

Tubular lattice members, particularly for foot bridges. Tension support systems Haunched or tapered sections Pre-cast bridge decks.

These innovative applications may be analysed using the principles of composite design, as explained in the following sections.

DESIGN OF COMPOSITE BEAMS - PLASTIC ANALYSIS

Principles of Composite Beam Design

Composite beams utilise the compression resistance of a concrete slab acting with a steel beam, whose cross-section is mainly in tension. The longitudinal forces are transferred between the steel and concrete by shear connectors that are welded to the top flange of the beam.

The proportion of the slab that is active with each beam is defined by its "effective width". For plastic design of the cross-section, the design strength of the concrete and steel are fully mobilised.

Composite design is optimised for simply supported beams where the elastic and plastic neutral axes lie at or close to the top flange of the beam. For continuous composite beams, the actions are reversed at the supports and the tensile forces in the slab are resisted by reinforcement.

For simply supported beams, composite design increases the bending resistance of the steel beam by $\circ \cdot$ to $1 \circ \cdot \%$ depending on the relative sizes of the beam and the slab. The increase in bending stiffness of a composite beam is even higher at $1 \circ \cdot$ to $1 \circ \cdot \%$, and so the span: depth ratio of composite beams can be in the range of $1 \circ \cdot \%$, rather than $1 \wedge 1 \sim 1 \circ \%$ for steel beams, reflecting their improved stiffness. Savings in steel weight can be $1 \circ \%$ relative to a steel beam for the same loading conditions.

Types of Composite Bridges

A composite bridge in its simplest form consists of a series of parallel I beams attached to a solid slab by shear connectors. The beams of I cross-section may be asymmetric in shape and are stabilised by cross-bracing to resist horizontal forces and to prevent buckling during construction. "Skew" bridges are of the same form except that the supports are not perpendicular to the beam span.

"Box" sections are efficient for curved beams or for cases resisting high torsional or horizontal forces. An open-topped box is stable in its completed form, but has to be braced during installation. A closed box is very stiff and can be installed by "launching" from one support. Both forms of box section can be analysed in bending as an equivalent asymmetric I section.

The generic cross-sections of composite bridges are presented in Figure), and many variants of this simple design are possible. For continuous bridges, some strengthening of the bottom flange may be required to resist negative (hogging) moments. Fabricated beams of variable cross-section along the span may also be designed efficiently.



(a) -beams - rolled or fabricated



Figure) Types of Composite BridgesEffective Slab Width

The proportion of the slab that is considered to be active with each beam is defined by its effective slab width. The phenomenon which determines the effective width of the slab is known as "shear lag", and is due to the higher strain experienced by the extreme fibres of the slab along the lines of principle compression. Due to this effect, the compressive stresses that can be resisted by the slab gradually reduce with increasing distance away from the centre-line of the beam.

The effective width may be taken for simply supported beams as beam span/ Σ for design at both the ultimate and serviceability limit states. This is the value adopted by BS o90+- Γ and EC Σ -). However, for bridge design to BS o Σ +-o, the effective width is taken as a function of the loading pattern and position in the slab, as defined by Table) for uniform loading.

For continuous beams, the effective width is taken as beam span/ Λ at the first internal support, which means that the reinforcement should be concentrated in a narrower zone than for the effective width at mid-span.

For edge beams, the effective width is taken as half the equivalent value for internal beams plus any slab overhang.

Slab width: Beam span B/L	Simply supported beams	Continuous beams	
	Mid-span	Mid-span	Support
•	<i>\</i> ,•	١,•	۱ ,•
•,•0	• ,٩٨	•,97	• ,٥Λ
•,)•	•,90	∙,∧٦	٠,٤١
•,7•	۰,∧۱	٠,٥٨	٠,٢٤
•,٣•	•,70	• ,۳۸	•,10
• •	•,0•	٠,٢٤	•,17
• ,0 •	• ,۳۸	•, T •	•,11

Table \Effective Widths for Composite Beams Subject to UniformLoading (to BS
ο٤··-ο)



(a) Cross-section showing effective slab width



(b) Planview of principal stresses in slab

Figure 7 Effective Width of the Slab and Lines of PrincipalCompression in a Composite Beam

Partial Factors

Partial factors are applied to loads and materials, and the partial factors depend on the level of safety implemented in the structural code. In buildings, partial factors are presented in BS 090+-1 and -7 and in Eurocodes 7-1 and 2-1. In bridges, partial factors are presented in BS 02+-7 and -0 and in Eurocodes 7-7 and 2-7.

Partial factors for loads are used to multiply the applied (working) loads, and partial factors for materials are used to reduce design strengths of the materials (i.e. steel and concrete and shear connectors), taking account of their variability in strength. These partial factors are presented in Table 7.

In BS $o \mathcal{E} \cdot \cdot \cdot \mathcal{T}$, the partial factors for dead loads take account of the variability in weight and dimensions of the components, which means that the surface material and concrete attract a higher partial factor than for steel. The partial factor for imposed load is taken as λ_0 for HA loading.

For materials strengths, the partial factor for steel is $1, \bullet 0$ and for concrete is 1, 0. An additional multiplication factor of $x_{1,1}$ is introduced in BS $0 \le \bullet - \%$ and -0 to take account of possible uncertainties in the design model (rather than the materials strengths). Therefore, the global factor of safety is in the region of $1, \Lambda$. Partial factors in the range of $1, \bullet$ to 1, % are also applied at the serviceability limit state (see below) – unlike in building design, where serviceability checks are based on working loads.

Load Type	Ultimate Limit State	Serviceability Limit State
Dead Loads:		
Concrete Steel Surface	1,10 1,•0 1,V0),•),•),T
Imposed Loads:		
HA HB	1,0 1,7	1,7 1,1

Table $\Upsilon(a)$ Partial Factors For Loads Υ_f to BS of Σ .

Table $\Upsilon(b)$ Partial Factors For Materials Υ_m to BSo2++-0

Material	Ultimate Limit State	Serviceability Limit State
Steel	1,•0	۱,•
Concrete	1,0	۱,٣

All partial factors for materials in BS $o \mathcal{E} \cdot \cdot \cdot \tilde{r}$ and -o are multiplied by),) to take account of uncertainties in the design model

Design Strength of Materials

The design strength of materials is obtained by dividing the characteristic strength by a partial safety factor (as above). For building design, the partial factor for steel is taken as $1, \cdot$ and for concrete as 1, 0.

For bridge design to BS $o \mathcal{E} \cdot \cdot -o$, the partial factors are $(, \cdot o x)$, and (, o x), for steel and concrete respectively (taking account of the multiplication factor of (,) noted above).

For shear connectors, a partial factor of λ , Yo is used to obtain their design resistance (or their design resistance is \cdot , Λ x characteristic resistance).

The design strengths of materials reduced by appropriate partial factors, as presented in Table Υ (a). These resistances are used in plastic design.

<u>Materials</u>	BS090+-γ and BS02++-0	Eurocode ٤
Steel	p _y /Υ _s Υ _s = ۱,• in BS090•-۳ or ۱,•ox۱,۱ in BS02••-۳	f _y (Υ _a = ١,• to ١,١)
Concrete	•,εοf _{cu} in BS090+-۳ •,ε•f _{cu} in BS0ε•0	$\cdot, \wedge of_c/\Upsilon_c$ ($\Upsilon_c = 1, o$) $f_c \approx \cdot, \wedge f_{cu}$
Reinforcement	∙ ,∧∨f _{yr}	f_{yr} / Υ ($\Upsilon_r = 1,10$)
Shear Connectors	∙,∧P _u	P_u / Υ_{sc} ($\Upsilon_{sc} = 1,70$)

Table $\Upsilon(a)$ Resistances of Materials to Various Codes

Table "(b) Resistances of Elements

Components	Resistances - BS 090+/02++	Eurocode ٤
Steel Section	R _s = •,Λ٦Α p _y in BS0٤••-0	$R_s = Af_y / \Upsilon_a$
Concerto Clab	$R_c = \star, \varepsilon o f_{cu} B_e D_s$ in BS090+-7	$R_{s} = \frac{\cdot . \Lambda o f_{c}}{1.0} B_{s} D_{s}$
Concrete Slab	= $\cdot, \Sigma f_{cu} B_e D_s$ in BSo $\Sigma \cdot \cdot - o$	
Reinforcement	$R_r = \cdot, \land \lor f_y A_r$	$R_r = \star, \land \lor f_y A_r$
Shear Connectors	$R_{sc} = \cdot AN_{sc} P_u$ $N_{sc} = number of shear connectors$	R _{sc} = ∙,∧N _{sc} P _u

Plastic versus Elastic Design

Plastic design is based on development of "plastic" stress blocks in the composite cross-section. Plastic stress blocks are rectangular, whereas elastic design uses stress blocks, which are linear elastic. Plastic design represents the maximum bending resistance which can be achieved for a given cross-section. The difference is best explained with reference to Figure ", in which the applied moment is gradually increased past the elastic limit.

In Figure Υ (a), elastic stresses and strains apply until either the concrete or steel reaches its design strength. From this point, elastic conditions do not hold. The strain in the bottom flange is ε_y when the applied stress equals f_{y} , which is the yield strength of steel.

In Figure Υ (b), the strain in the bottom flange increases to Υ_{ε_y} , and a proportion of the area of the web, (approximately half), reaches its design strength. The bending resistance of the composite section increases by o to $1 \cdot \%$.

In Figure Υ (c), the strain in the bottom flange increases to o_{ϵ_y} and most of the web has deformed plastically in tension. Also the parabolic stress-strain curve for concrete is almost fully developed. In this case, the bending resistance of the composite section has reached close to 90% of its theoretical plastic resistance.

In Figure Υ (d), the theoretical "plastic" stress blocks are developed at very high strain. This is clearly an idealised case, but it simplifies the analysis of the cross-section in bending. It is accepted that some strain hardening may occur in the bottom flange, which makes the use of plastic stress blocks an acceptable basis of design.



Figure[®] Elastic Through to Plastic Behaviour of a Composite Beam

Plastic stress blocks therefore represent the case of uniform design stresses existing throughout the material. It is first necessary to calculate the depth of the plastic neutral axis, which represents the line at which tension and compression forces are equal. The bending resistance is established by taking moments of the centre of each stress block around the plastic neutral axis.

In mathematical terms, the plastic neutral axis (PNA) depth, y_p below the top of the slab is given by equating the integral of the rectangular stress blocks above and below the PNA:

$$\int_{o}^{y_{p}} \sigma A = \int_{y_{p}}^{h} \sigma A$$

where σ is its design strength of the material, A is the cross-sectional area of each component, and h is the composite beam depth (beam and slab).

The plastic bending resistance is given by taking the "first moment of area" of each plastic stress block around y_p , according to :

$$M_{p_0} = \int \sigma A(y - y_p)$$

where y is the distance of the centre of each stress block, A, measured from the top of the slab.

Elastic stresses are based on linear elastic strains. For elastic design, the concrete element is transformed into equivalent steel element by dividing the cross-sectional area of the concrete slab by the modular ratio (Elastic modulus of steel : Elastic modulus of concrete – see later).

The elastic neutral axis depth (ENA), y_e of the composite section is obtained by taking moments of each element around an arbitrary position (normally the top of the slab) as follows:

$$y_e = \frac{\int yA}{\int A}$$

The elastic bending resistance is obtained from the "second moment of area" of each elastic stress block around y_e , plus the second moment of area of the web of the beam about its own axis. The second moment of area is expressed as follows:

$$I_{xx} = \int (y - y_e)^T A + t d^T / T$$

The second moment of area of the steel flanges and the slab about their own axes can generally be ignored.

The elastic bending resistance, based on the limiting stress in the bottom flange, is given by:

 $M_{_{e^{\Box}}} = f_{_{d}} I_{_{xx}} /(h - y_{_{e}})$ where f_{d} is the design strength of steel.

Section Classification

The section classification determines whether plastic or elastic design of the crosssection may be used. For simply supported composite beams, the top flange is fully restrained and does not suffer local or lateral buckling. The web is almost fully in tension, and so in most cases, the composite beam may be treated as Class ((plastic).

For continuous beams, the classification of the bottom flange and lower part of the web are important. The flange proportions and the depth of web in compression must satisfy the Class) or 7 limits in order that plastic design is used. If not, then elastic design should be used.

Generally in bridge design, elastic moments should be used to determine the moments acting at the ultimate limit state. Plastic section analysis may be used for Class Υ sections in negative bending by ignoring the ineffective portions of the web in compression.

In the construction stage, the top flange of the steel is unrestrained, and so it is necessary to check the stability of the beam between lateral restraints. Generally, elastic design of the beam is used in the construction stage in which case, it should satisfy the Class Γ limits for the cross-section.

Plastic Design of Asymmetric Steel Section

Consider the general case of a simply supported asymmetric steel beam as shown in Figure Σ . The plastic bending resistance can be established by first determining the plastic neutral axis (PNA) from the "equal area" principle, as shown below. Then, moments may be taken of the force acting centre of each rectangular stress block around the PNA, as follows:

Plastic Neutral Axis Depth, yp

$$R_{ft} + R_{w} \left(\frac{y_{p}}{d} \right) = R_{fb} + R_{w} \left(\frac{d - y_{p}}{d} \right)$$
$$y_{p} = \frac{d}{\gamma} \frac{\left(\frac{R_{fb}}{fb} - \frac{R_{fb}}{ft} + \frac{R_{w}}{w} \right)}{R_{w}}$$

Plastic Moment Capacity

$$M_{p_{D}} = R_{f_{D}} (D - y_{p}) + R_{f_{t}}y_{p} + \frac{R_{w}y_{p}}{Td} + \frac{R_{w}(d - y_{p})^{t}}{Td}$$



Figure ٤ Plastic Analysis of Asymmetric Steel Beam

Example of Plastic Resistance of Asymmetric Steel Beam



y_p = Plastic Neutral Axis Depth (PNA)

Tensile Resistance of Bottom Flange

 $R_{fb} = A_{fb}f_{yd} = \text{ ``o } x \text{ ``\cdot } x \text{ ``v } x \text{ ``v } kN$

Tensile Resistance of Top Flange

 $R_{ft} = A_{ft}f_{vd} = 10 x \tau \cdot \cdot x \tau \cdot v x 1 \cdot \tau = 971 kN$

Tensile Resistance of Web

$$R_w = dt_w f_{yd} = (\wedge \cdot \cdot - \cdot \cdot) \times \wedge \circ \times \forall \cdot \vee \times \wedge \cdot \cdot = \forall \circ \cdot \cdot kN$$

Check if: $R_{ft} + R_w \ge R_{fb}$; PNA Lies in Web

PNA:

 $\mathbf{y}_{\mathbf{p}} = \underbrace{\frac{(\wedge \cdot -}{\cdot \boldsymbol{\Sigma} \cdot \boldsymbol{\Sigma}}}_{\boldsymbol{\Sigma}} \times \underbrace{\frac{(\nabla^{\boldsymbol{\nabla} \cdot} - \boldsymbol{\gamma} \cdot \boldsymbol{\Sigma})}{\cdot \boldsymbol{\Sigma}}}_{\boldsymbol{\Sigma} \cdot \boldsymbol{\Sigma} \cdot \boldsymbol{\Sigma}} = \underbrace{\boldsymbol{\Sigma} \cdot \boldsymbol{\Sigma}}_{\boldsymbol{\Sigma} \cdot \boldsymbol{\Sigma} \cdot \boldsymbol{\Sigma}} \quad \text{in web}$

Plastic Moment Capacity

$$\mathbf{M}_{\mathbf{P}_{0}} = \mathsf{Y}^{\mathsf{W}} \cdot \mathsf{Y} \times (\mathsf{Y}^{\mathsf{V}} \cdot + \mathsf{Y}^{\mathsf{T}} - \circ \mathsf{W}^{\mathsf{V}}) \times \mathsf{Y}^{-\mathsf{W}} + \mathsf{Y}^{\mathsf{Y}} \times \circ \mathsf{W}^{\mathsf{Y}} \times \mathsf{Y}^{\mathsf{V}}^{-\mathsf{W}}$$

$$\overset{\bullet}{\mathsf{T}}_{\mathsf{O}} \underbrace{\overset{\bullet}{\overset{\bullet}}}_{\mathsf{T}} \times \underbrace{\overset{\bullet}{\overset{\bullet}}}_{\mathsf{T}} \overset{\mathsf{O}}{\overset{\bullet}}_{\mathsf{T}} \overset{\mathsf{O}}{\overset{\bullet}}_{\mathsf{T}} \times \underbrace{\overset{\bullet}{\overset{\bullet}}}_{\mathsf{T}} \times \underbrace{\overset{\bullet}{\overset{\bullet}}}_{\mathsf{O}} \overset{\mathsf{O}}{\overset{\bullet}}_{\mathsf{T}} \overset{\mathsf{O}}{\overset{\mathsf{O}}}_{\mathsf{T}} \overset{\mathsf{O}}{\overset{\mathsf{O}}}_{\mathsf{T}}} \overset{\mathsf{O}}{\overset{\mathsf{O}}}_{\mathsf{T}} \overset{\mathsf{O}}{\overset{\mathsf{O}}}_{\mathsf{T}} \overset{\mathsf{O}}{\overset{\mathsf{O}}}_{\mathsf{O}} \overset{\mathsf{O}}{\overset{\mathsf{O}}}_{\mathsf{T}} \overset{\mathsf{O}}{\overset{\mathsf{O}}}_{\mathsf{T}} \overset{\mathsf{O}}{\overset{\mathsf{O}}}_{\mathsf{O}} \overset{\mathsf{O}}{\overset{\mathsf{O}}}_{\mathsf{O}} \overset{\mathsf{O}}{\overset{\mathsf{O}}} \overset{\mathsf{O}} \overset{\mathsf{O}}} \overset{\mathsf{O}}{\overset{\mathsf{O}}} \overset{\mathsf{O}}{\overset{\mathsf{O$$

= ••• + ٤٩٤ + ٦٤٦ + ١٢٢ = ١٨١٩ kNm

Plastic Design of Composite Section

The analysis of plastic design may be repeated for a composite cross-section. Three cases of positions of the plastic neutral axis exist depending on the relative resistances of the concrete slab and steel section, which are established, as follows:

1. Plastic neutral axis in the concrete slab

In this case, the compressive resistance of the concrete slab, R_c exceeds the tensile resistance of the steel section, R_s , and so the plastic neutral axis lies in the slab to a depth of:

$$y_{p} = \frac{R_{s}}{R_{c}} D_{s}$$

where D_s is the slab depth and R_s is the tensile resistance of the steel section = $R_{ft} + R_{fb} + R_w$ and R_c is the compression resistance of the slab.

Taking moments about the centre of compression in the slab leads to a plastic bending resistance of:

$$M_{pI} = R_{ft} (D_s - \cdot .oy_p) + R_{fb} (D + D_s - \cdot .oy_p)$$
$$+ R_w (D_s + \cdot .oD - \cdot .oy_p)$$

(Note: it is normal practice to ignore the finite thickness of the steel flanges in this formula).



Case $: R_c > R_s$:

Figure • Plastic Stress Blocks for Plastic Neutral Axis in the Slab

^{*}. Plastic neutral axis in the top flange of the beam

In this case, $R_s > R_c$, but a further condition of $R_s - R_{ft} > R_{ft} + R_c$ applies in order for the PNA to lie in the top flange (R_{ft} = tensile resistance of top flange).

Taking moments about the centre of the top flange leads to a bending resistance of:

$$M_{p_{0}} = R_{c} \frac{D_{s}}{\gamma} + R_{fb}D + R_{w} \frac{D}{\gamma}$$

The plastic neutral axis depth in the top flange and may be taken as equal to $\mathsf{D}_{\mathsf{s}}.$

Case T: $R_s > R_c$ and $R_{fb} + R_w < R_c + R_{ft}$:



Figure V Plastic Stress Blocks for Plastic Neutral Axis in the Top Flange

". Plastic neutral axis in the web of the beam

In this case, $R_s - R_{ff} < R_{ft} + R_c$, so that some of the web acts in compression. It is convenient to define a further term y_w which is the depth of web in compression, whose depth is given by:

$$y_{w} = \frac{d}{\tau} (R_{fb} + R_{w} - R_{c} - R_{ft}) / R_{w}$$
$$y_{p} = y_{w} + D_{s}$$

where d is the web depth

The plastic neutral axis depth is given by $y_p = y_w + D_s + t_f$, where t_f is the thickness of the top flange.

This is clearly a more complex case, and only applies for heavy beams or for highly asymmetric beams. The plastic bending resistance is given as:

$$M_{pl} = R_{c} (y_{w} + \cdot .0 D_{s}) + R_{ft} y_{w} + R_{fb} (D-y_{w})$$
$$+ R_{w} \left[\frac{y_{w}}{\gamma_{d}}^{T} + \frac{(d-y_{w})^{T}}{\gamma_{d}} \right]$$

Case
$$\tilde{r}$$
: $R_{fb} + R_w \ge R_c + R_{ft}$ and $R_{fb} < R_c + R_{ft} + R_w$



When the plastic neutral axis lies deep in the web, there is a possibility that the web may be Class T rather than Class T, but this occurs rarely except when the bottom flange area is γ or more times the top flange area.

In this case, the effective depth of the web in compression is taken as $\Upsilon \Lambda t \varepsilon$, where $\varepsilon =$ $(TVo/f_v)^{\circ}$ to BS 090. The same approach is used for beams subject to negative bending where a proportion of the web in compression is ineffective -see later.

Example of Plastic Resistance of Composite Beam



Compressive Resistance of Concrete Slab

$$R_{c} = \cdot, \xi f_{cu} B_{e} D_{s} / 1, 1 = \cdot, \xi \times \tau \cdot x \tau \cdot x \cdot x 10 \cdot x + \tau' / 1, 1 = \xi 9 \cdot 9 kN$$

Tensile Resistance of Bottom Flange

$$R_{fb} = f_{vd} A_{fb}/\gamma_a = 70 \times 7.4 \times 7.4 \times 1.4^{-7} = 77.4 \times 10^{-7}$$

Tensile Resistance of Top Flange

$$R_{ft} = f_{yd} A_{ft}/\gamma_a = 10 \text{ x T + x T + V x } + 7^{-7} = 971 \text{ kNTensile}$$

Resistance of Web

$$R_w = f_{vd} A_w / \gamma_a = (\Lambda \cdot \cdot - \Sigma \cdot) \times \log \pi \cdot \vee \times 1 \cdot \cdot^{\tau} = \pi \circ \cdot \cdot kN$$

Total Tensile Resistance of Steel Section:

 $R_s = R_{fb} + R_{ft} + R_w = \text{TT+T} + \text{9T} + \text{To++} = \text{7VTT} \text{ kN}$

R_s > R_c, So Plastic Neutral Axis Lies in Steel Section

Example of Plastic Analysis – Continued

Find Plastic Neutral Axis (PNA) Position

Check if: $R_c + R_{ft} \ge R_{fb} + R_w$ for PNA in Top Flange

 $29 \cdot 9 + 97 \geq 77 \cdot 7 + 70 \cdot \cdot$

 $\mathrm{div} + \mathrm{div} + \mathrm$

So Plastic Neutral Axis Just Lies in Top Flange

Take Moments About Centre of Top Flange (for simplicity):



Compare to Plastic Resistance of Steel Beam

$$M_{p_0} = \Lambda R k N m$$

Increase in Bending Resistance Due to Composite Action

= 97%